# Vector Space

**Definition** (Vector Space). A set is called a vector space, if it is equipped with an addition operation and a scalar multiplication operation such that the following properties are satisfied for all and .

1. and
2. There is an element such that [*Neutral element*]
3. There is an element called such that
4. and [*Distributive property*]
5. [*Associative property*]

The elements are called *vectors*.

**Remark**

A “vector” multiplication with is not defined. Theoretically, we could define an element-wise multiplication, such that with . This “array multiplication” is common to many programming languages but makes mathematically limited sense using the standard rules for matrix multiplication, since the dimensions of the vectors do not match.

Only the following multiplications for vectors are defined: (outer product), (inner/scalar/dot product).

**Example**

There are many different vector spaces that will pop up in data science:

1. Most important: , the with () that consists of tuples of real numbers, defined as follows:
   * Addition: for all
   * Multiplication by scalars: for all .
2. is a vector space with
3. The space of complex vectors (with the standard definition of addition of complex numbers) is less prominent but may occur as well.
4. As important: *Spaces of functions!* The set of all functions from some set into the real numbers form a vector space as well. Such a space may be used to model a set of decision functions that we want to build to predict an output for given data . If we want to predict more than just one number for a data point we consider the space of functions , and those functions form a vector space as well.